



# Configuration of $p$ -Cycles in WDM Networks with Partial Wavelength Conversion

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**Abstract.** The  $p$ -cycle concept offers a capacity-efficient and rapid protection mechanism for mesh-restorable networks. This work investigates the configuration of span protecting  $p$ -cycles in wavelength division multiplexing (WDM) networks with limited wavelength conversion. An important point of view is the relation between the costs associated with the number of required wavelength converters, and the protection capacity-efficiency achieved. We formulate mathematical models and solve the respective optimization problems for a pan-European network as a test-case. An interesting finding is that the total number of converters required for the network as a whole can be greatly reduced, with only a small increase in spare capacity for protection by a strategy of associating wavelength converters with the access points between a pure wavelength path (WP) working layer and a set of pure WP  $p$ -cycle protection structures.

**Keywords:**  $p$ -cycles, wavelength conversion, WDM, integer linear programs

## 1 Introduction

The  $p$ -cycle concept has become an attractive method for span protection in wavelength division multiplexing (WDM) transport networks, since it combines the benefits of both ring-based protection and mesh-based restoration [4]. An important point introduced by WDM networks is the routing and wavelength assignment problem [6]. Without any (wavelength) converters, each demand path must have the same wavelength from origin to destination node. In WDM networks without wavelength conversion, situations can occur where there still exists a significant total number of free channels, but one common free wavelength throughout a path cannot be found. Changing the wavelength along the lightpath can prevent this blocking phenomenon. The deployment of wavelength converters thus increases flexibility, however, it also adds high hardware costs.

We investigate the  $p$ -cycle protection concept in WDM networks with the idea of requiring as few wavelength converters as possible, without significant penalty due to such wavelength-blocking effects. We develop new approaches for an efficient configuration and focus on the two aspects of protection capacity-efficiency and the number of required wavelength converters.

This article is organized as follows. The remaining part of this section gives definitions for some important terms. Section 2 summarizes the  $p$ -cycle concept for span protection. In Section 3, we introduce architectures for  $p$ -cycles in WDM networks with partially converting nodes, and describe mathematical models to find optimal  $p$ -cycles for these architectures. Section 4 presents the architectures' performance in terms of efficiency and converter usage. In Section 5, we draw several conclusions.

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A *link* denotes an individual wavelength-channel between two adjacent nodes. A *span* stands for the set of all links between two adjacent nodes. We call a wavelength converter simply a *converter*. A *path* (in general) is an end-to-end connection described by a sequence of links and nodes. A *wavelength path (WP)* is a specific type of path that consists of links with the same wavelength and thus does not require any wavelength converters. A *virtual wavelength path (VWP)* includes a wavelength converter at its traversing nodes and can have different link-wavelengths. A *WP network* has no wavelength conversion capabilities at all, and at the opposite extreme a *VWP network* has wavelength conversion for every input-output wavelength at every node. While Schupke et al. [6] considers and compares  $p$ -cycles in pure WP and pure VWP networks, we will now be looking at networks that require some wavelength converters at certain locations, but much less than a pure VWP network. The motivation is saving costs by requiring as few converters as possible, but without incurring the significant capacity penalty of a pure WP network. The idea, novel in this article, is how to achieve this cost-minimizing balance in the particular case of a WDM network based on the  $p$ -cycle concept for protection.

## 2 The Concept of $p$ -Cycles

The  $p$ -cycle concept, first presented in Grover and Stamatelakis [3], is a recent strategy to recover from network failures. It provides a protection mechanism for spans and for transiting traffic through failed nodes.  $p$ -Cycles can be described as pre-configured closed protection paths in a mesh network. They are formed in the spare capacity of a two-connected mesh network. Like rings, the protection capacity is pre-connected in advance of any failure and the protection switching is very fast and simple. Unlike rings, however,  $p$ -cycles support shortest-path routing of working paths over a network and protect spans that straddle the ring-like cycle of protection capacity, as well as failures on the cycle itself. It turns out that the latter, apparently simple difference leads to an enormous improvement in capacity-efficiency. By the term “capacity-efficiency” we denote the ratio between the capacity reserved for  $p$ -cycles on all spans and the total capacity needed for working paths.

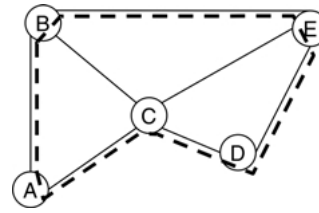


Fig. 1. Simple mesh network with one  $p$ -cycle shown for illustration.

$p$ -Cycle protected networks are essentially as capacity-efficient as span-restorable mesh networks.

To explain  $p$ -cycles, we concentrate on the most frequent outage scenario, the failure of a single span. This happens when, for example, a backhoe cuts a span while digging the ground. It is assumed that all fibers of the span are broken and data transmission in both directions is no longer possible. All the network connections which use the failed span are affected.

In the following figures we refer to bidirectional links that consist of a pair of unidirectional links in opposite directions. Fig. 1 depicts the topology of a small mesh-restorable network with five nodes and seven spans. The dashed line represents a  $p$ -cycle of a single unit of protection capacity. Like a self-healing ring, the  $p$ -cycle is able to protect an on-cycle span failure as shown in Fig. 2a. Beyond that however, a  $p$ -cycle is also able to protect straddling spans. These are spans whose end nodes lie on the  $p$ -cycle but are not contained in the ring. For a straddling span failure, like a cut of the span B-C in Fig. 2b, a  $p$ -cycle in fact provides protection for two working paths on the failed span. Such circumstance contributes to the high capacity-efficiency of  $p$ -cycles compared with known ring protection concepts, like the unidirectional path-switched ring (UPSR) and the bidirectional line-switched ring (BLSR).

From a graph-theoretical point of view, the particular  $p$ -cycle depicted is a Hamiltonian cycle,

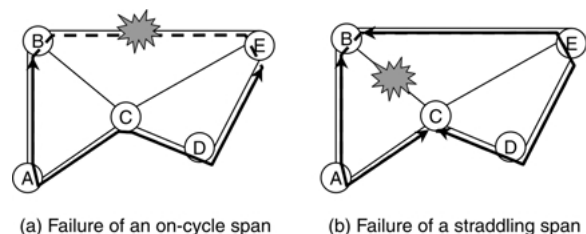


Fig. 2.  $p$ -Cycle protection paths (solid).

i.e., it traverses through all the nodes of the network. This means that this one protection structure can protect all the spans of the network. It offers one protection path for five on-cycle spans and two protection paths for each of the two straddling spans. Its capacity is not dedicated to recover from one particular span failure, but can be shared for single span outages at every on-cycle or straddling span.

More generally, in a network with arbitrary mixtures of demands between the nodes, the most efficient solution is achieved by the deployment of multiple  $p$ -cycles, not all of which will be based on Hamiltonian cycles but will depend more generally on the routing of the working paths. To assure 100% restorability of any single span failure in the network, each span must be provided with at least as many protection paths as the number of working links it bears. When a network span fails, all of the affected working links on that span have a protection path to restore the connection.

The protection capacity needed for the  $p$ -cycles is reserved in advance of any network failure, but shared over all on-cycle and straddling links associated with the  $p$ -cycle. When a span cut occurs, the resulting signal loss is detected by both end nodes of the failed span. Only these two nodes have to perform any real-time action and switch each concerned working link to its protection path. By pre-configuration, the assignment of the  $p$ -cycle protection paths to working links for each span failure is known.

The  $p$ -cycle concept combines the merit of the two basic protection strategies so far. On one hand, it allows short recovery times of about 50–150 ms known from protection ring structures. On the other hand it has been shown in several pieces of work now that under appropriate design methods, it offers the capacity-efficiency which is essentially as high as that of a span-restorable mesh network (see, for example, Grover and Doucette [2], Grover and Stamatelakis [3], and Schupke et al. [6]).

Most often the logical design of a  $p$ -cycle protected network can be divided into two steps. In a first step, the routing of working paths for each node pair is effected without direct regard for survivability considerations at that stage. This may typically be simply a matter of shortest-path routing, or flow-leveled shortest path routing as in Schupke et al. [6]. Secondly, if it is assumed that all channels of modular WDM systems are turned up on each span,  $p$ -cycles

are then formed in the remaining available capacity, or more generally, spare wavelength channels are added only as needed to form a minimal-cost set of  $p$ -cycles that support 100% restorability of the working capacity total on each span. The two-step approach is also called the non-joint design method. In a joint approach [2], the routing of working paths and the placement of spare channels and logical formation of  $p$ -cycles are found simultaneously, usually with the goal of minimal total cost.

$p$ -Cycles can be configured by a network management system [4,6], or by distributed self-organization [3,4]. We focus on the former case using off-line calculations. Note that, as the latter case yields slightly sub-optimal results, the off-line approach represents a best case for the distributed approach.

### 3 $p$ -Cycles in WDM Networks with Partial Conversion

WDM networks add the aspect of wavelength assignment to the  $p$ -cycle protection concept. Working and  $p$ -cycle links may have different transmission wavelengths. It is not sufficient to provide only enough  $p$ -cycle protection paths for the working links but one also has to consider the wavelengths of the working paths failed in any given scenario and the wavelength(s) on which a  $p$ -cycle is established. If the protection path for a working link is allocated at a different wavelength, the wavelength must be converted to access the  $p$ -cycle in case of a failure. Otherwise, all working link and protection path arrangements must be coordinated to be at the same wavelengths [6].

In nodes with partial wavelength conversion, only a limited number of incoming lightpaths can change their wavelength towards the outgoing link. Fig. 3 depicts the share-per-node architecture offering a converter pool of  $C$  converters [5]. If there is no wavelength conversion required, an incoming lightpath will be directed to the appropriate output port of an outgoing fiber or to the local access. Other-wise the lightpath can pass through an available converter in the converter pool. For reference, note that in a fully equipped WP network there are no converters, and in a VWP network each node has at least  $M \cdot K$  converters.

In the following subsections, we consider two main architectures to provide  $p$ -cycles in WDM networks

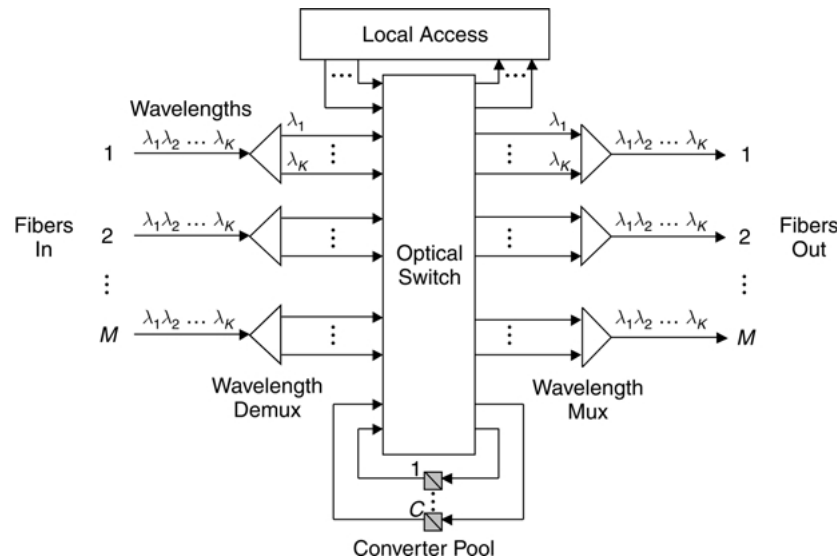


Fig. 3. Node architecture using partial conversion.

with partial wavelength conversion. One uses WP  $p$ -cycles with converters used to access them, and the other has VWP  $p$ -cycles.

A central idea is to provide for protection without requiring a set of  $p$ -cycles dedicated to every wavelength, but while using as few converters as possible overall, it may be efficient to associate converters only with the  $p$ -cycle access points, leaving working paths to be implemented in a WP manner. Although the converters are available for working paths, we assume the converters are used for protection paths only. Working paths are then WP with a fixed wavelength. One reason for this assumption is that protection paths typically need more flexibility in the wavelength selection to be able to protect many working wavelengths. Moreover, since a failed working link on the path is replaced by a longer protection path, the converters can aid in the regeneration of the signal of affected client paths, which are enlarged in physical length. Note that this assumption on the working traffic makes the

following architectural alternatives also realizable by nodal configurations other than the shared converter pool node architecture.

In this basic approach, after detecting a span failure the adjacent nodes switch the working link to the protection path on the  $p$ -cycle and, if necessary, convert the wavelength of the working link to that of the  $p$ -cycle. As the details for both actions are known in advance, it seems possible to perform these two steps at the same time, since a parallel switching and wavelength conversion function helps to speed up the failure recovery.

Thus, we distinguish between two layers in the network; one in which the working paths are routed and one in which the  $p$ -cycles are provided for protection of the former. Fig. 4 summarizes the possible options for the working path layer, the  $p$ -cycle protection layer, and the interface between the two, to identify the particular combinations considered here. To minimize converters as a general aim, we will assume the working path layer to be

WP			VWP	Protection paths ( $p$ -cycles)
(i) no [6]	(ii) if required	(iii) always	(iv) only for straddling spans	Converters to access $p$ -cycles
WP				Working paths

Fig. 4. Considered architectures to provide wavelength conversion.

comprised entirely of WPs which, of course, use no converters. The  $p$ -cycles may be formed either as WP or VWP structures, however.

If working links access WP  $p$ -cycles in failure situations without passing a converter, we obtain the left-most case (i) of a WP network protected by WP  $p$ -cycles, which was considered already in Schupke et al. [6]. Option (ii) in Fig. 4 shows the possibility of a WP working layer accessing WP  $p$ -cycles with converters only deployed selectively as needed to change from working to  $p$ -cycle wavelength (see also Fig. 5). This allows failed working links to access a  $p$ -cycle without wavelength blocking.

Such a design would, however, be rather “brittle” in the sense that the selectively placed converters are really only assured to avoid blocking for a specific demand pattern. A more systematic and robust architecture from a standpoint of flexibility to demand pattern, which can still considerably reduce wavelength converters, is to provide that every failed working signal passes through a converter to access a  $p$ -cycle. Thus (iii) is the case where both  $p$ -cycles and working paths are WP structures, but matching between wavelengths is provided at every  $p$ -cycle access point. This separates the problem into two WP assignment problems, one for working paths and one for the  $p$ -cycles. Note as well that due to the regeneration capability of converters for paths which are made longer during protection switching, the signal degradation is kept low. Effectively, the working path is terminated at the end-points of the working link.

The last alternative in Fig. 4, is subtly, but significantly, different. Instead of using converters to access WP  $p$ -cycles, we consider that the  $p$ -cycles

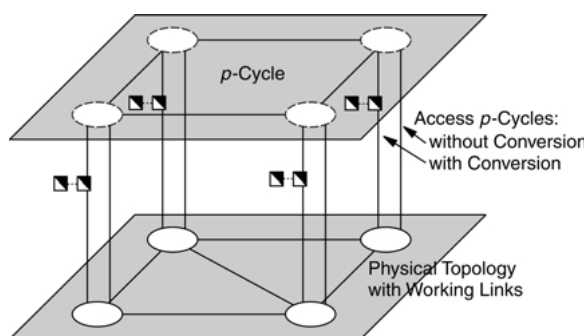


Fig. 5. If needed, converters translate the failed working signals to the required wavelength to access a  $p$ -cycle for protection.

themselves would be entirely VWP structures where every link of a  $p$ -cycle has end-points on wavelength converters (see Fig. 6). These converters can both interface any WP working path into a  $p$ -cycle as needed for protection and also provide wavelength-agility in series along the  $p$ -cycle to permit  $p$ -cycle formation with any available spare channel, regardless of wavelength. Thus (iv) represents a pure VWP  $p$ -cycle layer design sitting over a pure WP working path layer. It is clear that with VWP  $p$ -cycles, we may introduce more converters in total if the same number of  $p$ -cycles is used as in the other cases, but on the other hand, by designing the VWP  $p$ -cycles we may tend to require fewer  $p$ -cycles in total, as well as less total capacity, so this scheme is also of interest to characterize.

Note that for a given topology, there are more distinct VWP  $p$ -cycle combinations that are possible than WP  $p$ -cycles. A  $p$ -cycle using converters partially on the path is another, more general option (not considered in this article) to prevent wavelength blocking in use of  $p$ -cycles. In effect the latter option is the sister of (ii) in Fig. 4 instead of selective use of converters to access WP  $p$ -cycles, it would represent selective use of converters to form VWP  $p$ -cycles. (Or, more precisely, VWP  $p$ -cycles formed of WP path segments.)

In Section 3.1 we shall discuss the mathematical model for WP  $p$ -cycles with converters used to access them, and in Section 3.2 the model using VWP  $p$ -cycles.

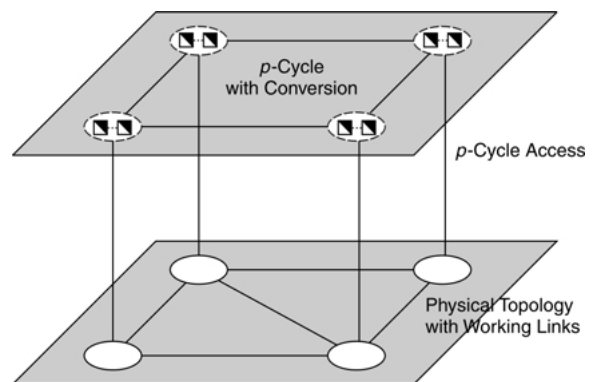


Fig. 6. Illustrating a VWP  $p$ -cycle where every  $p$ -cycle node has wavelength conversion to adapt re-routed working signals to use the available spare wavelengths of each hop in the logical  $p$ -cycle.

### 3.1 WP $p$ -Cycles with WP-working and Converters at $p$ -Cycle Access

To reiterate, in this case wavelength conversion is available for working paths to access  $p$ -cycles, but thereafter the entire path through the  $p$ -cycle for any restoration purpose must adhere to the one common wavelength of the  $p$ -cycle. We assume that there is a converter pool at every node and aim to minimize the total number of converters. Converters, however, are used only to translate failed working paths on any possible wavelength, on to and out of path segments on desired  $p$ -cycles used for their protection. Thus in effect, the concept is of a WP working layer, accessing a set of WP  $p$ -cycles. Converters are used only for the mapping between the layers, to adapt fixed working wavelengths of failed paths onto the fixed working wavelengths of  $p$ -cycles which are shared over many different failure scenarios.

#### 3.1.1 Basic (Non-joint) Design model

In this subsection the optimization is non-joint; i.e., it is based on working WPs given as inputs. The mathematical model describes the optimization problem for the total cost of capacity needed for  $p$ -cycles and converters needed to access the  $p$ -cycles under all different failure scenarios. We use a bidirectional orientation where each span is understood to offer the same number of lightpaths in both directions. Links are consistently formed out of two unidirectional lightpaths in opposite directions on a span.

Sets:

**N**: WDM nodes  $n$

**S**: Bidirectional spans (usually indexed by  $j$ , indexed by  $i$  for failed spans)

**K**: Available wavelengths  $\lambda$  per unidirectional fiber

**P**: Eligible bidirectional  $p$ -cycles  $p$

Parameters:

$m_j \in \mathbb{R}^+$ : Length metrics for each span  $j$

$\tau_{n,j} \in \{0, 1\}$ : Indicates whether span  $j$  is incident to node  $n$  ( $= 1$ ) or not ( $= 0$ )

$f \in \{1, 2, \dots\}$ : Number of pairs of two reverse unidirectional fibers in the spans

$w_{j,\lambda} \in \{0, 1, 2, \dots\}$ : Number of bidirectional working links on span  $j$  at wavelength  $\lambda$

$\delta_{p,j} \in \{0, 1\}$ : Indicates whether  $p$ -cycle  $p$  lies on span  $j$  ( $= 1$ ) or not ( $= 0$ )

$x_{p,i} \in \{0, 1, 2\}$ : Number of protection paths an instance of  $p$ -cycle  $p$  provides for failed span  $i$  ( $= 0$

for neither on-cycle nor straddling span,  $= 1$  for an on-cycle span,  $= 2$  for a straddling span)

$\alpha \in \mathbb{R}^+$ : Weight factor for the cost of inserting a converter

Variables:

$s_{p,\lambda} \in \{0, 1, 2, \dots\}$ : Number of unit capacity  $p$ -cycles  $p$  using wavelength  $\lambda$

$c_{i,\lambda}^{\text{spanWave}} \in \{0, 2, 4, \dots\}$ : Number of converters needed per end node of failed span  $i$  for wavelength  $\lambda$

$c_i^{\text{span}} \in \{0, 2, 4, \dots\}$ : Number of converters needed per end node of failed span  $i$  for all wavelengths

$c_n \in \{0, 2, 4, \dots\}$ : Number of converters needed at node  $n$

Objective:

$$\text{minimize} \left( \sum_{p \in \mathbf{P}} \sum_{\lambda \in \mathbf{K}} s_{p,\lambda} \cdot \sum_{j \in \mathbf{S}} \delta_{p,j} \cdot m_j + \alpha \cdot \sum_{n \in \mathbf{N}} c_n \right), \quad (1)$$

subject to:

$$\sum_{p \in \mathbf{P}} \sum_{\lambda \in \mathbf{K}} s_{p,\lambda} \cdot x_{p,i} \geq \sum_{\lambda \in \mathbf{K}} w_{i,\lambda}, \quad \forall i \in \mathbf{S} \quad (2)$$

$$w_{j,\lambda} + \sum_{p \in \mathbf{P}} s_{p,\lambda} \cdot \delta_{p,j} \leq f, \quad \forall j \in \mathbf{S}, \quad \forall \lambda \in \mathbf{K} \quad (3)$$

$$c_{i,\lambda}^{\text{spanWave}} \geq 2 \cdot \left( w_{i,\lambda} - \sum_{p \in \mathbf{P}} s_{p,\lambda} \cdot x_{p,i} \right), \quad \forall i \in \mathbf{S}, \quad \forall \lambda \in \mathbf{K} \quad (4)$$

$$c_i^{\text{span}} = \sum_{\lambda \in \mathbf{K}} c_{i,\lambda}^{\text{spanWave}}, \quad \forall i \in \mathbf{S} \quad (5)$$

$$c_n \geq \tau_{n,i} \cdot c_i^{\text{span}}, \quad \forall n \in \mathbf{N}, \quad \forall i \in \mathbf{S}. \quad (6)$$

The objective (1) minimizes the protection capacity and the number of converters multiplied by a weight factor that represents the cost of wavelength converters relative to an additional wavelength channel of capacity.  $p$ -Cycle resources are measured by the length of all allocated  $p$ -cycle links. The weight factor  $\alpha$  can thus be interpreted as saying that a converter has costs equal to  $\alpha$  kilometers of a deployed wavelength channel. The constraints (2) ensure that the sum of working links on any span are protected by at least an equal amount of protection paths offered by  $p$ -cycles, i.e., any single span failure can be absorbed. Constraints (3) prevent each span from using more links at one wavelength than the number of fiber pairs.

Constraints (4) calculate for each span failure the number of converters needed at each of the span's end nodes to protect all working links on the span at a certain wavelength.  $w_{i,\lambda}$  is the number of all bidirectional working links with wavelength  $\lambda$  on the failed span  $i$ .  $s_{p,\lambda} \cdot x_{p,i}$  is the number of protection paths at wavelength  $\lambda$  offered by  $p$ -cycles  $p$  with wavelength  $\lambda$ .  $w_{i,\lambda} - \sum_{p \in \mathbf{P}} s_{p,\lambda} \cdot x_{p,i}$  therefore calculates the number of remaining bidirectional working links at wavelength  $\lambda$  on the failed span  $i$  that are not protected by  $p$ -cycles at the same wavelength. If this term is  $> 0$ , the result multiplied by 2 is the number of converters which is needed at each of the two end nodes of the failed span  $i$  to retune the wavelength  $\lambda$  to be able to use a  $p$ -cycle with a different wavelength. The factor of two takes into account that two, only for one direction usable converters, are needed for one bidirectional path.

Equalities (5) calculate for each span failure the number of converters needed at each of the span's end nodes to protect all working links on the span at any wavelength. Constraints (6) calculate for each node the number of converters that is needed to protect all working links on any adjacent span at any wavelength.  $\tau_{n,i} \cdot c_i^{\text{span}}$  calculates the necessary number of converters in any node  $n$  for that node and any possible incident failed span.  $c_n$  has to be greater than or equal to this term to assure that the highest appearing number of converters needed for any incident span failure is assigned. The minimum number of converters per node is obtained by the objective.

If failed working links should always pass a converter before accessing a  $p$ -cycle, the model simplifies. Effectively, the number of needed converters in a node is determined by the maximum number of adjacent working links. Then constraints (4) to (6) and the second addend in the objective can be dropped.

### 3.1.2 Joint Optimization of Working Paths and Protection Configuration

Joint optimization takes both the working routing and wavelength assignment into account. In doing so it may choose a more capacity-efficient configuration of working paths plus  $p$ -cycles, and further minimize the converter costs in one step. The mathematical model is based on the former one for the non-joint optimization (Section 3.1.2) with extensions as needed for joint optimization.

The determination of the working capacity is based on a path approach. For each demand relation  $d$

between two nodes, eligible routes  $r_d$  are calculated in advance of the optimization process. These represent only possible working routes, not a decision about the working route. The latter is left to be decided as part of the joint optimization.

*Additional Sets:*

**D:** Bidirectional demand relations  $d$  between nodes  
**R<sub>d</sub>:** Eligible bidirectional routes  $r_d$  for demand relation  $d$

*Additional Parameters:*

$u_d \in \{0, 1, 2, \dots\}$ : Number of capacity units for relation  $d$   
 $\varepsilon_{j,d,r_d} \in \{0, 1\}$ : Indicates whether route  $r_d$  for relation  $d$  lies on span  $j$  ( $= 1$ ) or not ( $= 0$ )

*Additional Variables:*

$w_{d,r_d,\lambda} \in \{0, 1, 2, \dots\}$ : Number of working paths on route  $r_d$  at wavelength  $\lambda$  for relation  $d$

*Objective:*

$$\begin{aligned} \text{minimize} & \left( \sum_{d \in \mathbf{D}} \sum_{r_d \in \mathbf{R}_d} \sum_{\lambda \in \mathbf{K}} w_{d,r_d,\lambda} \cdot \sum_{j \in \mathbf{S}} \varepsilon_{j,d,r_d} \cdot m_j \right. \\ & \left. + \sum_{p \in \mathbf{P}} \sum_{\lambda \in \mathbf{K}} s_{p,\lambda} \cdot \sum_{j \in \mathbf{S}} \delta_{p,j} \cdot m_j + \alpha \cdot \sum_{n \in \mathbf{N}} c_n \right), \end{aligned} \quad (7)$$

subject to:

$$\sum_{r_d \in \mathbf{R}_d} \sum_{\lambda \in \mathbf{K}} w_{d,r_d,\lambda} = u_d, \quad \forall d \in \mathbf{D} \quad (8)$$

$$\begin{aligned} \sum_{p \in \mathbf{P}} \sum_{\lambda \in \mathbf{K}} s_{p,\lambda} \cdot x_{p,i} \geq \sum_{d \in \mathbf{D}} \sum_{r_d \in \mathbf{R}_d} \sum_{\lambda \in \mathbf{K}} w_{d,r_d,\lambda} \cdot \varepsilon_{i,d,r_d}, \\ \forall i \in \mathbf{S} \end{aligned} \quad (9)$$

$$\begin{aligned} \sum_{d \in \mathbf{D}} \sum_{r_d \in \mathbf{R}_d} w_{d,r_d,\lambda} \cdot \varepsilon_{j,d,r_d} + \sum_{p \in \mathbf{P}} s_{p,\lambda} \cdot \delta_{p,j} \leq f, \quad \forall j \in \mathbf{S}, \\ \forall \lambda \in \mathbf{K} \end{aligned} \quad (10)$$

$$\begin{aligned} c_{i,\lambda}^{\text{spanWave}} \geq 2 \cdot \left( \sum_{d \in \mathbf{D}} \sum_{r_d \in \mathbf{R}_d} w_{d,r_d,\lambda} \cdot \varepsilon_{i,d,r_d} \right. \\ \left. - \sum_{p \in \mathbf{P}} s_{p,\lambda} \cdot x_{p,i} \right), \quad \forall i \in \mathbf{S}, \quad \forall \lambda \in \mathbf{K} \end{aligned} \quad (11)$$

$$c_i^{\text{span}} = \sum_{\lambda \in \mathbf{K}} c_{i,\lambda}^{\text{spanWave}}, \quad \forall i \in \mathbf{S} \quad (12)$$

$$c_n \geq \tau_{n,i} \cdot c_i^{\text{span}}, \quad \forall n \in \mathbf{N}, \quad \forall i \in \mathbf{S}. \quad (13)$$

In comparison to the non-joint problem, the objective function is augmented by the working capacity. It minimizes the total length of all working paths and  $p$ -cycles plus the total costs for all inserted converters. An additional constraint is added via equations (8). It ensures that for each demand relation, the number of provided working paths equals the number of demand wavelengths. The other constraints correspond to the ones from the non-joint optimization, and are just modified as far as the different notation of the working links is concerned. If failed working links should always pass a converter before accessing a  $p$ -cycle, the model simplifies by dropping the subtrahend in (11).

### 3.2 VWP $p$ -Cycles and WP Working Layer

In the VWP  $p$ -cycles approach, each node on a  $p$ -cycle has a converter available for protected signals that traverse it. In the design problem this is thought to give two advantages at once: it allows that any free wavelength on a span can be assigned to become part of any logical  $p$ -cycle that the design requires. In addition it allows that a working link on any wavelength can pass through a converter to access any  $p$ -cycle in a failure situation.

#### 3.2.1 Basic (Non-joint) Design Model

The mathematical model for this approach is based on the same parameters, sets, and variables as before. The variable for the amount of unit capacity  $p$ -cycles  $s_p$  now needs to be indexed over all eligible cycles only. When a  $p$ -cycle is chosen, we do not determine the wavelengths it has at its different links. In practice, this can easily be done in a separate step after the determination of the  $p$ -cycles. The model can thus be kept simple.

*Variables:*

$s_p \in \{0, 1, 2, \dots\}$ : Number of unit capacity  $p$ -cycles  $p$

*Objective:*

$$\text{minimize} \left( \sum_{p \in \mathbf{P}} s_p \cdot \sum_{j \in \mathbf{S}} \delta_{p,j} \cdot m_j + \alpha \cdot \left( 2 \cdot \sum_{p \in \mathbf{P}} s_p \cdot \sum_{j \in \mathbf{S}} \delta_{p,j} + 2 \cdot \sum_{p \in \mathbf{P}} s_p \cdot \sum_{j \in \mathbf{S}} (x_{p,j} - \delta_{p,j}) \right) \right), \quad (14)$$

subject to:

$$\sum_{p \in \mathbf{P}} s_p \cdot x_{p,i} \geq \sum_{\lambda \in \mathbf{K}} w_{i,\lambda}, \quad \forall i \in \mathbf{S} \quad (15)$$

$$\sum_{\lambda \in \mathbf{K}} w_{j,\lambda} + \sum_{p \in \mathbf{P}} s_p \cdot \delta_{p,j} \leq f \cdot |\mathbf{K}|, \quad \forall j \in \mathbf{S}. \quad (16)$$

The objective function (14) minimizes the  $p$ -cycle capacity in terms of link-km plus the total costs for all converters. The number of converters is not chosen freely but depends on the allocated  $p$ -cycles. A  $p$ -cycle has wavelength conversion functionality at each traversed node for the end-points of on-cycle spans, and additionally for straddling spans. Although, at a node, the converters for straddling spans can be shared by disjoint straddling spans terminating at that node, we do not consider this in order to be concise. A factor of two is introduced, because bidirectional  $p$ -cycles need two converters per link. Constraints (15) guarantee that the working links on each span are protected by at least an equal number of  $p$ -cycle protection paths. Because of the converters at each  $p$ -cycle node, the individual wavelengths of working links on a span do not matter. Constraints (16) make sure that the number of working links plus  $p$ -cycle links does not exceed the available wavelength capacity at any span.

#### 3.2.2 Joint Optimization of Working Paths and $p$ -Cycles

The mathematical model for the joint optimization in the case of VWP  $p$ -cycles is also based on the previous definitions, with the following refinements.

*Variables:*

$s_p \in \{0, 1, 2, \dots\}$ : Number of unit capacity  $p$ -cycles  $p$

$w_{d,r_d,\lambda} \in \{0, 1, 2, \dots\}$ : Number of working paths on route  $r_d$  at wavelength  $\lambda$  for relation  $d$

*Objective:*

$$\begin{aligned} & \left( \sum_{d \in \mathbf{D}} \sum_{r_d \in \mathbf{R}_d} \sum_{\lambda \in \mathbf{K}} w_{d,r_d,\lambda} \cdot \sum_{j \in \mathbf{S}} \varepsilon_{j,d,r_d} \cdot m_j \right. \\ & \quad \left. + \sum_{p \in \mathbf{P}} s_p \cdot \sum_{j \in \mathbf{S}} \delta_{p,j} \cdot m_j \right. \\ \text{minimize} & \quad \left. + \alpha \cdot \left( 2 \cdot \sum_{p \in \mathbf{P}} s_p \cdot \sum_{j \in \mathbf{S}} \delta_{p,j} \right. \right. \\ & \quad \left. \left. + 2 \cdot \sum_{p \in \mathbf{P}} s_p \cdot \sum_{j \in \mathbf{S}} (x_{p,j} - \delta_{p,j}) \right) \right) \end{aligned} \quad (17)$$

subject to:

$$\sum_{r_d \in \mathbf{R}_d} \sum_{\lambda \in \mathbf{K}} w_{d,r_d,\lambda} = u_d, \quad \forall d \in \mathbf{D} \quad (18)$$

$$\sum_{p \in \mathbf{P}} s_p \cdot x_{p,i} \geq \sum_{d \in \mathbf{D}} \sum_{r_d \in \mathbf{R}_d} \sum_{\lambda \in \mathbf{K}} w_{d,r_d,\lambda} \cdot \varepsilon_{i,d,r_d}, \quad \forall i \in \mathbf{S} \quad (19)$$

$$\sum_{d \in \mathbf{D}} \sum_{r_d \in \mathbf{R}_d} w_{d,r_d,\lambda} \cdot \varepsilon_{j,d,r_d} \leq f, \quad \forall j \in \mathbf{S}, \quad \forall \lambda \in \mathbf{K} \quad (20)$$

$$\sum_{d \in \mathbf{D}} \sum_{r_d \in \mathbf{R}_d} \sum_{\lambda \in \mathbf{K}} w_{d,r_d,\lambda} \cdot \varepsilon_{j,d,r_d} + \sum_{p \in \mathbf{P}} s_p \cdot \delta_{p,j} \leq f \cdot |\mathbf{K}|, \quad \forall j \in \mathbf{S}. \quad (21)$$

The objective (17) minimizes the total working and protection resources plus the costs for all wavelength converters. The used network capacity is determined by the length of all deployed links. Constraints (18) guarantee that each demand relation is satisfied. The constraints (19) ensure a sufficient protection of the working links against single span failures. As each  $p$ -cycle has converters at each traversed node, working links at any wavelength can use the cycle. Thus the quantity of working links per span stipulates the number of protection paths the  $p$ -cycle configuration has to provide. A span with more working links at the same wavelength than the number of fibers is prevented by constraints (20). As any free wavelength can be assigned to  $p$ -cycles at traversed spans, constraints (21) need only to ensure that the number of links for both working and protection does not exceed the capacity on the span.

## 4 Numerical Results Comparing Schemes

In a case study we examine the performance of the architectures using the optimization models of the previous section. We compare the proposed approaches for  $p$ -cycles for partial wavelength conversion to each other and to the results for pure WP and VWP networks [6]. For these comparisons we use the pan-European COST239 network [1]. It is an optical core network which connects 11 cities with 26 spans. With an average nodal degree of 4.73, the network is quite densely meshed. In order to derive lightpath granularity from the demand matrix in Batchelor et al. [1], the values for the given demand

transmission rates are divided by 2.5 Gbit/s. Furthermore we adapt two unsymmetrical demand pairs by taking the higher value such that a consistent bidirectional demand matrix is created. The resulting offered load is 176 bidirectional lightpath demands. We assume one (counter-directional) fiber-pair per span and 32 wavelengths per fiber, and no wavelength conversion as the default case. The total capacity of the network sums up to  $32 * 15\,045$  link-km = 481 440 link-km. For  $p$ -cycles, we deploy simple cycles, i.e., closed paths traversing through a span or a node at most once.

For the non-joint design cases, the allocation of  $p$ -cycles depends on the number of deployed working links on each span in the network and on their wavelength arising from prior routing and wavelength assignment for the working paths. A shortest path algorithm with span-length as metric calculates the working path for each node pair. If a shortest route is not unique, we choose the one with the least number of hops, i.e., the smallest number of spans. Concerning the wavelength assignment, we apply a ‘‘First-Fit’’ mechanism.

For the joint optimization we take all shortest routes as the set of eligible routes. As the working capacity is calculated as the sum of the working routes’ mileage, this cost is the same for both non-joint and joint optimization. We calculate a value of 137 170 link-km for the working capacity.

We abbreviate the investigated architectures:

- netWP: WP network (WP working, WP  $p$ -cycles, no converters)
- wWPpWPaR: working WP, protection WP, access with conversion if required
- wWPpWPaF: working WP, protection WP, access with full conversion
- wWPpVWP: working WP, protection VWP (and access with full conversion)
- netVWP: VWP network (VWP working and VWP protection)

### 4.1 Cost Comparison

In Fig. 7 we compare the total design cost (i.e., the objective value of the optimization models) including both converter and capacity costs, as the cost of one converter is varied relative to the cost of a defined channel-distance of transmission capacity. This is done for each of the different architectures. It was deemed appropriate not to assume a specific cost for wavelength

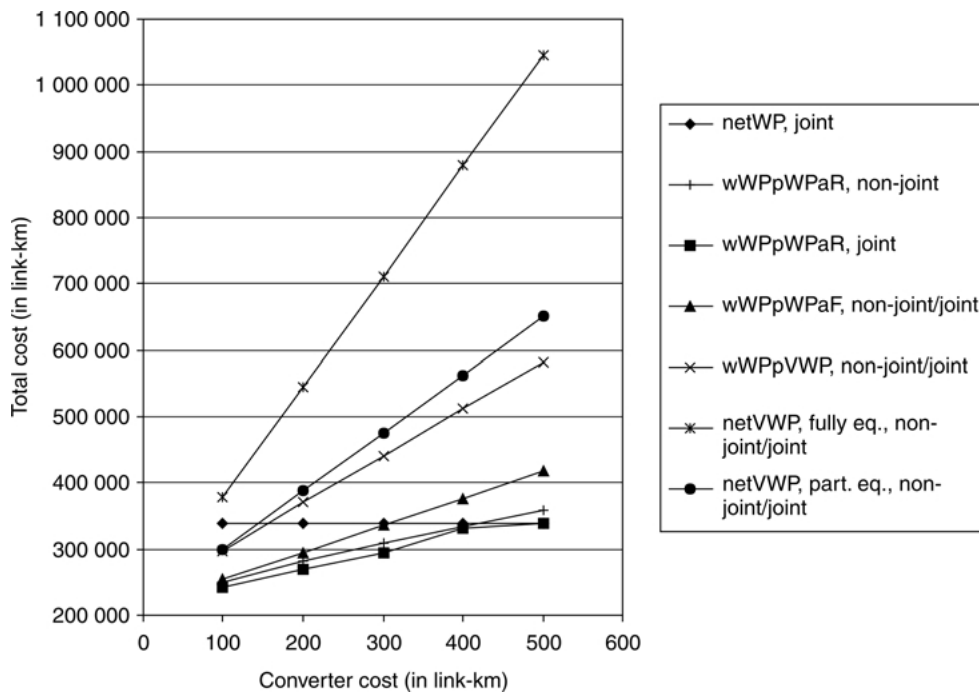


Fig. 7. Total cost (for converters, working capacity, and protection capacity in the network) over the cost for a single converter.

conversion as this could vary greatly with supplier, technology, time, and volumes. This is why the converter cost is expressed in “equivalent link-km.” In other words the absolute converter cost is expressed in terms of the number of link-km’s of transmission that would have the same actual cost. Except for  $wWPpWPpAR$ , only the results obtained by joint optimization are shown. In the non-joint optimization,  $netWP$  has no feasible solution (see also Section 4.2). The architectures  $netVWP$  and  $wWPpVWP$  have the same cost values as in joint and non-joint optimization. Small cost additions of 4% and 0.8% are on average required in the non-joint optimization for  $wWPpWPpAR$  and  $wWPpWPpAF$ , respectively. In summary, it is worth remarking that, whenever at least some conversion is allowed in the network, a more restricted working routing (e.g., as in the non-joint approach) may incur no or only small additional cost.

If we assume a fully equipped VWP network ( $netVWP$ , fully eq.), we cope with significantly high total cost. Only for very low converter cost, it comes near the cost domain of the other architectures. A partially equipped VWP network ( $netVWP$ , part. eq.), where converters are employed for used (working and protection) links only, reduces the total cost by 1/3, but it still reaches high cost levels.

For the joint optimization, as converters are not used in  $netWP$ , the cost stays at a constant value. The  $wWPpVWP$  curve is slightly lower than the  $netVWP$ , partially equipped curve. Both outperform  $netWP$  only for very low converter cost, e.g., at 100 link-km converter cost, the total cost is 13% less.

The architecture  $wWPpWPpAF$  is up to 75% better for low converter cost (100 link-km converter cost) than  $netWP$ , and breaks even to it for converter cost of about 300 link-km.

The  $wWPpWPpAR$  architecture generates even less cost. Only for the non-joint optimization and at high converter cost it exceeds  $netWP$  cost, e.g., at 500 link-km converter cost the total cost is 6% more. The joint optimization for  $wWPpWPpAR$  yields the best cost values. For low converter cost values, the curve is close to the non-joint counterpart, while for higher converter cost values, it approaches the  $netWP$  curve. Note that, due to the complexity of the joint  $wWPpWPpAR$  optimization, we could only compute sub-optimal results.

## 4.2 Capacity-Efficiency

The diagram in Fig. 8 shows for the five architectures the capacity-efficiency after joint and non-joint

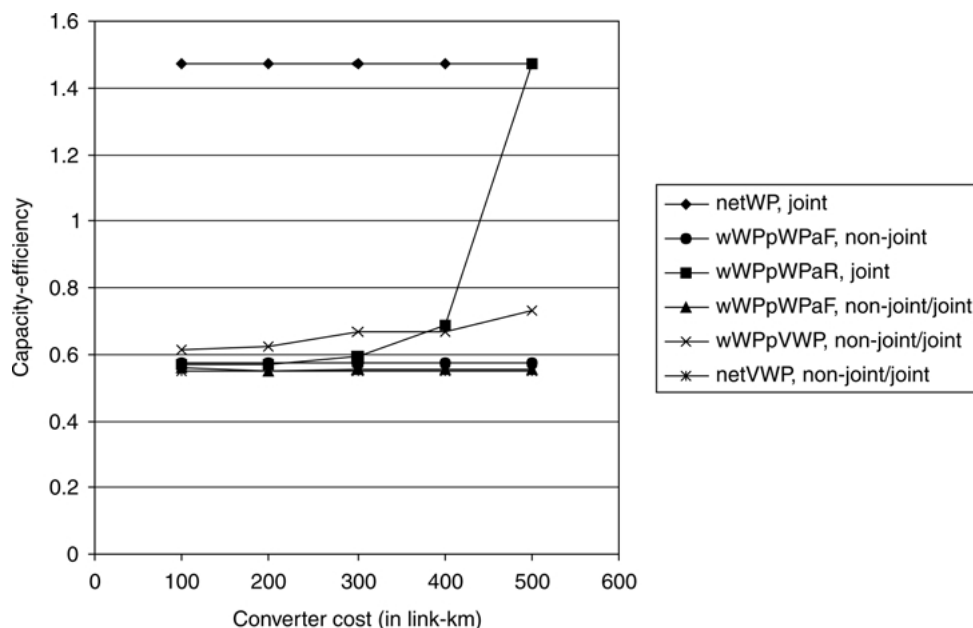


Fig. 8. Capacity-efficiency (protection capacity/working capacity) versus relative cost of converters (joint and non-joint optimization).

optimization. Capacity-efficiency here refers to the distance-weighted ratio of protection to working capacity over the network as a whole. The curves are drawn as a function of the cost of one converter expressed in terms of an equivalent wavelength-channel cost. The lower bound value of 55% for the efficiency of any architecture is given by full conversion ( $\text{netVWP}$ ), while a feasible solution of the non-joint problem with no conversion ( $\text{netWP}$ , non-joint) does not even exist. (It would if spare capacity could be added as needed, but under the assumptions here of one fiber pair per span at exactly 32 wavelengths per fiber, infeasibility is possible). For the joint optimization, the working paths can change and this degree of freedom already helps to find a solution with no converters ( $\text{netWP}$ , joint). As one fiber-pair per span makes wavelength conversion very restrictive,  $\text{netWP}$  requires a high level for the protection capacity of about 1.5 times the working capacity.

For  $\text{wWPpWPaR}$ , the efficiency increases from 58% for converter-costs at 100 link-km to the  $\text{netWP}$  efficiency level at 500 link-km. Therefore, converters will be replaced by more protection capacity, i.e., longer  $p$ -cycles are taken to avoid wavelength blocking. A similar behavior holds for the efficiency of  $\text{wWPpVWP}$ , which climbs from 61% to 73% for the two cost values. For high cost values, both

architectures require up to one third more protection capacity than  $\text{netVWP}$ , while for low cost values, they are comparable in efficiency. The outcome for  $\text{wWPpVWP}$  is equal, since with  $p$ -cycles having full wavelength conversion, adapting working paths cannot aid in efficiency here.

Because of the fixed working capacity, the number of converters is constant for  $\text{wWPpWPaF}$ , and, as a result, the cycles do not change as the costs change. Thus the efficiency takes the constant value of 57%, close to the  $\text{netVWP}$  case.

### 4.3 Deployment of Converters

Fig. 9 depicts how many converters are deployed in the optimal selection as a function of the cost of one converter device. Only for  $\text{wWPpWPaR}$ , the results are different in the non-joint and joint optimizations.

For  $\text{netVWP}$ , we need 1664 and 876 converters for the fully and partially equipped versions, respectively. For the entire network,  $\text{netVWP}$  thereby requires the most devices.

Only 242 to 334 converters are deployed for  $\text{wWPpWPaR}$  in the non-joint optimization. For low cost,  $\text{wWPpWPaR}$  needs 248 to 270 converters in the joint optimization, and no converters for cost of 500 link-km.

As in  $\text{wWPpVWP}$   $p$ -cycles have to have full conversion, on average 718 converters are needed.

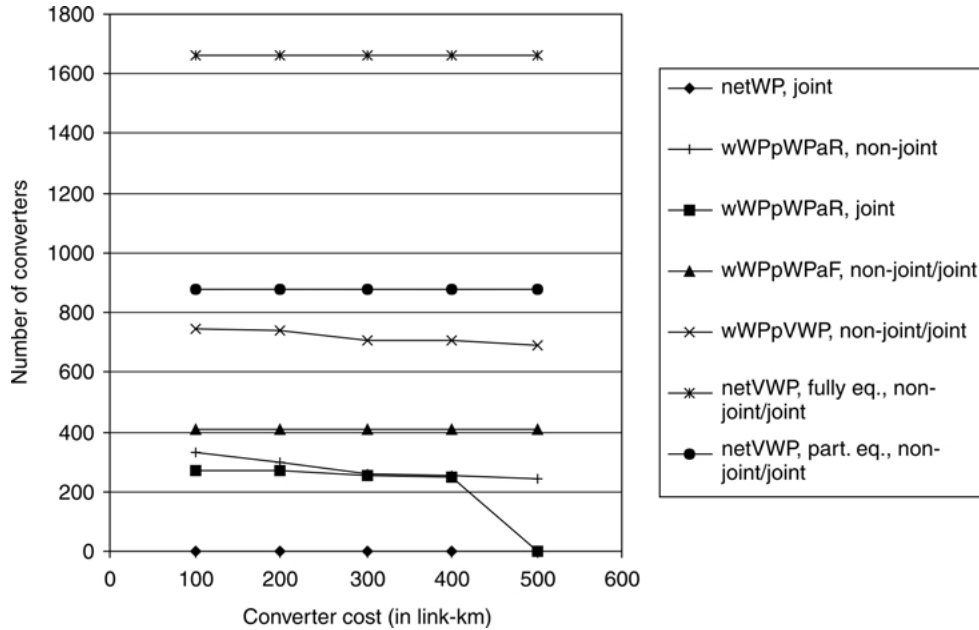


Fig. 9. Number of deployed converters over cost of one converter.

By also taking the efficiency results of the previous sections into account, this architecture does not offer much benefit in both efficiency and converter consumption compared with *netVWP*.

The *wWPpWPpPaF* architecture requires less than a half of the number of converters the *netVWP* architecture needs, while the efficiency is only slightly higher. For a fair cost comparison, however, the cost of fixed lasers in *netVWP* has to be opposed to the higher costs of tunable lasers in the architectures with partial conversion.

## 5 Conclusions

This article deals with the configuration of span protecting  $p$ -cycles in wavelength division multiplexing networks deploying partial wavelength conversion in the nodes. We presented two basic architectures where the converters are made available for  $p$ -cycles. The first one makes the converters available for failed WP working links when accessing  $p$ -cycles that are themselves WP structures. The other approach forms VWP  $p$ -cycles directly, using full conversion at each node of each  $p$ -cycle. Failed WP working link signals are then adapted into the VWP

$p$ -cycles at their span-failure end-nodes and rerouted through the VWP  $p$ -cycles. The VWP  $p$ -cycles are formed before failure, allowing use of any spare link to become part of a  $p$ -cycle. For these architectures we formulated optimization models and used them for a case study of a pan-European network, where we made the comparison to respective results for networks with no and full wavelength conversion capabilities.

Numerical results indicated that an architecture, which locates conversion at the access points of WP  $p$ -cycles offers high efficiency. Compared with a network with full conversion, a high amount of converters can be saved.

Fig. 10 summarizes the results of this article. It evaluates the capacity-efficiency and the converter consumption. Wavelength flexibility measures how easy working paths with arbitrary wavelengths can be protected by a  $p$ -cycle. Furthermore, it is indicated if tunable lasers (which have higher cost than fixed lasers) at the access converters are needed, and if the protection path along the  $p$ -cycle is terminated (for regeneration and monitoring) by converters at the switching nodes. Note that if required, real time tunability of the converters is only a requirement at the  $p$ -cycle access interfaces from the WP working layer, because  $p$ -cycles themselves are either WP in nature or, if VWP, are nonetheless structures that are

	Conversion at $p$ -cycle access	Protection capacity efficiency	Converters	Wavelength flexibility	Tunable lasers required	Termination of protection path
WP Working paths and WP $p$ -cycles with conversion at access as indicated	No	very low	No	very low	n/a	no
	Selective	high	very few	low	yes	some paths
	Mandatory	very high	few	high	yes	yes
WP Working paths, VWP $p$ -cycles and full conversion at access		high	more	high	yes	yes
Full VWP Network		very high	most	n/a	no	yes

Fig. 10. Evaluation of the architectures.

completely configured before failure, so any wavelength conversions required are static re-assignments of wavelengths.

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